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A REFINEMENT OF THE FISHER'S EQUATION

Abstract
Fisher's equation, or the equation of exchange, relates the money supply to the price levels, the monetary dynamics, and the economic activity. This equation, in its most cited form, shows imprecisions when it comes to the dimensional analysis of the quantities it relates. A need for clarifying the interpretation of these quantities and for facilitating the application of this equation has motivated this endeavour. In this work, first the Fisher's equation has been analysed dimensionally, and certain difficulties for its correct interpretation have been shown. Then, a basic exposition of the operation of an economic system has been given, together with an analysis of the dynamics of trade in a money based economy. It has been shown that for each participant in an economy there is a monetary holding function, which is composed of mathematically simple elementary monetary holding functions. These elementary monetary holding functions arise naturally from trading transactions. One important consequence of this analysis is the discovery that the essential function of the money in an economy is that of memory of contributions to the economy by the participants in trade. In the sequel, a new form of the equation relating the supply of money to the

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trade activity and level of prices has been derived. The derivation has been performed from the underlying mathematical principles, and with attention being paid to the dimensions of the involved quantities. With some well justified simplification, a refined form of the Fisher's equation has been produced. A facilitated application of the equation in its new form has been shown through a brief analysis of the phenomenon of monetary inflation, and the influence of the money supply on prices and overall economic activity.

Key words: *quantity of money, Fisher's equation, monetary economics.*

JEL classification: B16, E31, E40

Introduction

Irving Fisher's equation of the quantity of money, in the original called the equation of exchange, is a brilliant mathematical expression relating the money supply in an economy to the level of prices and to the economic activity. It is as beautiful as it is useful. Nevertheless, its application sometimes meets certain difficulties, arising from apparent imprecision in its formulation. Namely, a better clarity is necessary as to the dimensions, i.e. the units of measurements, of the quantities used in this equation in its usual form. This task has been endeavoured in this work. Section 1 presents a brief dimensional analysis of the original Fisher's equation, together with some critical remarks, and a possible direction towards a resolution of the problems arising. In Section 2, an elementary description of functioning of an economy, from a monetary point of view, has been presented as a sequence of time distributed transactions happening between the agents. In Section 3, the dynamics of monetary holdings of the economic agents has been exposed. An important commentary issuing from this section is that the role of money in an economy is that of the memory of past activities of the participants. In Section 4, a step by step derivation of a new formulation of the Fisher's equation of exchange has been presented. At its very outset, a rigorous mathematical interpretation of the monetary activities is followed, which produces a somewhat complex expression. After a few steps of simplification, a concise form of the equation has been derived. In order to illustrate the convenience of this equation in its new form, its application to a brief analysis of the phenomenon of monetary inflation is presented in the Section 5.

1. Dimensional Analysis of the Fisher's Equation

The equation linking the quantity of money in an economy to the level of prices, and the velocities of circulation of money and of traded goods, has been proposed by Irving Fisher. In the original called the equation of exchange, it is a brilliant mathematical expression relating the supply of money in an economy to the level of prices and to the economic activity. It has been stated in various forms, but one form often met in the literature is:

$$MV = PQ \tag{1}$$

where M is the total quantity of money available, V is, so called, the velocity of money, P is the level of prices, and where Q is the measure of economic activity, also called the aggregate transactions.

Despite its brilliance, this equation suffers from certain imprecision. This becomes apparent when its dimensional analysis in the physical sense is attempted. To illustrate these shortcomings, let us first assume that the dimension of money, taken as a quasi physical quantity, is dollar [\$]. Further on, the dimension of the velocity of money V is dollars per second $[\$s^{-1}]$. For the right hand side of the equation, let us assume that the product exchanged in transactions is measured in kilograms$[kg]$. Then, the prices P will have dimension of dollars per kilogram, $[\$kg^{-1}]$, whereas the measure of economic activity, or the aggregate transactions Q is expressed by kilograms $[kg]$. Then, overall, the dimensions of the original equations are:

$$MV[\$^2s^{-1}] = PQ[\$] \tag{2}$$

Clearly, there is a dimensional mismatch between the two sides of the equality. In order to remedy this situation, the dimension of the velocity of money V may be taken to be that of frequency, i.e. the reciprocal seconds $[s^{-1}]$. Even though this may not be quite an acceptable interpretation for ordinary physical quantities, this is a positive step towards the correct dimensions of the sides of the money exchange equation. However, the dimensional analysis of the Fisher's equation still shows a mismatch:

$$MV[\$s^{-1}] = PQ[\$] \tag{3}$$

In a further attempt to reconcile the dimensions of the left hand side to those of the right hand side of the equation, the aggregate transactions Q will take the meaning of the velocity of transactions. In the illustrative case introduced above, this will mean that the dimension of the velocity of the aggregate transactions Q is $[kgs^{-1}]$. Only with this interpretation, the Fisher's equation becomes dimensionally balanced:

$$MV[\$s^{-1}] = PQ[\$s^{-1}] \tag{4}$$

Fisher's equation, whilst it is undoubtedly correct in its concept, leaves in its expression certain possibilities for misinterpretation. For this reason, it may be advantageous to transform this equation into another form. In its new form, the money exchange equation may be more easily and more widely applied. The new proposed form of the equation, together with its derivation, is given in the next three sections of this work. Finally, the equation of the quantity of money is applied to a brief analysis of the phenomenon of monetary inflation.

2. Dynamics of Money Movement in an Economy

In this section, the functioning of an economy as a system will be defined, together with some other related terms. This is necessary to performe in order to streamline the subsequent derivation of the reformulated quantity of money equation. As a consequence of this description, it will become apparent that the role of money in an economy is to record past activities of the agents.

In this work, the economy, or alternatively, the market, is taken to be a system in which its members, here called the agents, can satisfy their needs by obtaining goods or services, and in which these goods and services can be provided by other agents. Agents may be people, or businesses, or any other entities that are capable of this behavior. In an economy, agents can perform economic transactions, that is, they can trade. During a transaction, two agents interact in order to perform a transaction. One of the agents of a transaction is the seller, and this agent provides goods or services that satisfy a need of the other agent. The other agent in a transaction is the buyer, and it compensates the seller with an agreed amount of money. The equation linking the quantity of money in an economy to the level of prices, and the velocities of circulation of money and of traded goods, has been proposed by Irving Fisher.

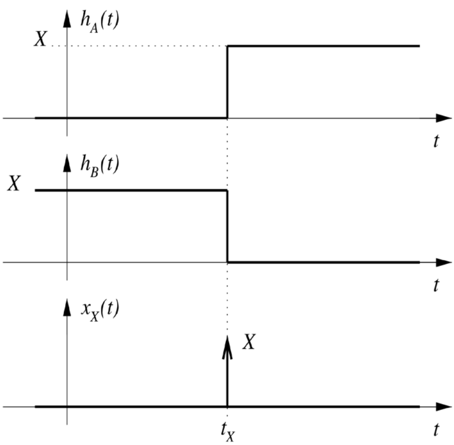

Figure 1. Time diagram of an economic transaction

The goods or services provided by the seller are needed by the buyer in some definite quantity, or to some definite extent, that may be measured in some appropriate units. Let this quantity be denoted by G. In order to transform the measure expressed by quantity G into money, it has to be multiplied by its agreed price P. This product is equal to the quantity of money exchanged in that transaction.

$$X = PG \tag{5}$$

Quantity X will be called the value of the transaction. A transaction may be described as a process in which some non monetary value is exchanged for the same monetary value.

Transactions in an economy happen at random times. It is, therefore, necessary to describe a transaction not only by its value, but also as an event taking place in time. Both quantities, the goods or services exchanged, and their prices, may vary with time, so these both of these quantities should be taken as functions of time, $G(t)$ and $P(t)$. On the other hand, the process of exchanging values in a transaction happens instantly. In other words, the time duration of a transaction is infinitely small. A transaction with these properties may be considered to be a discrete event, occurring at some random time instant t_X. In order to represent mathematically such an event, one can recourse to the Dirac pulse $\delta(t)$, which has infinitely short duration, $\delta(t) = 0$, for all $t \neq 0$, yet, it has unitary area, that is to say, its total integral value is one, $\int_{-\infty}^{+\infty} \delta(t)dt = 1$. It is also assumed that $P(t)$ and $G(t)$ are continuous, and that $G(t)$ has at least a first order derivative. A transaction happening at time instant t_X, with time functions for price $P(t)$, and for quantity $G(t)$, is represented by

$$x(t) = P(t)G(t)\delta(t - t_X) = P(t_X)G(t_X)\delta(t - t_X) \tag{6}$$

Here, the sampling property of the Dirac's delta pulse $\delta(t)$ has been explicitly used. An example of the dynamics of transactions is illustrated in Fig 1. The transaction represented in that figure is between agents A and , where agent A is the seller, and agent B who is the buyer. Together with the time diagram of the transaction, the figure shows changes of the respective monetary holdings, which will be explained in the following section of this article.

At this moment, the dimensionality of the quantities announced here should be mentioned. The dimension of the money exchanged in a transaction X is, of course, money, e.g. dollar [$]. The dimension of price P is money per unit of goods or services, e.g. dollar per kilogram $[\$kg^{-1}]$, whereas the dimension of the quantity of goods or services exchanged G is in the units in which these are measured, e.g. kilograms $[kg]$. However, the sampling property of the Dirac's pulse requires that the dimension of a transaction, viewed as a time dependent quantity (t), be in monetary units per time, e.g. dollars per second $[\$s^{-1}]$, i.e. the dimension of $x(t)$ is that of money velocity. This will be show more clearly in the next section.

3. Money Circulation and Monetary Holdings

For each agent in an economy, there is a time dependent quantity of money currently available to that agent, i.e. the money owned by that agent, (t). This quantity represents the agent's monetary holding. Monetary holding changes only at time instances when transactions occur, so that at each transaction in which that agent is the seller, the value of the transaction is added to the monetary holding, while at each transaction in which the agent appears to the buyer, the value of the transaction is deducted from the agent's monetary holding.

A particular form of a monetary holding is the elementary monetary holding. Elementary monetary holding, $h(t)$, has constant non-zero value between the time of acquisition, t_A, and the time of relinquishment, t_R. Outside time interval $[t_A, t_R]$, the value of the elementary monetary holding is identical to zero. Analytically, it may be represented as

$$h(t) = P(t)G(t)\big(\delta(t - t_A) - \delta(t - t_R)\big) \tag{7}$$

The time diagram of an elementary monetary holding is presented in Fig. 2. The figure presents time diagram of transactions amongst three agents, A, B and C. At instant t_A, there is the interaction between agent A, the seller, and agent B, the buyer. Goods or services in quantity $G_1(t_A)$ at price $P_1(t_A)$ are being provided by A to B in exchange for amount of money $= P_1(t_A)G_1(t_A)$. At that instant, agent acquires an elementary monetary holding, denoted by $h_A(t)$ in the figure. This elementary monetary holding $h_A(t)$ ends by the transaction at time instant t_R, in which agent A, now the buyer, acquires goods or services from agent C, who is the seller in this transaction. The goods or services are received by the buyer in quantity $G_2(t_R)$ and at price $P_2(t_R)$, in exchange for an amount of money $X = P_2(t_R)G_2(t_R)$. In this case, it is the same amount as the one exchanged at that elementary monetary holding's acquisition time t_A. The duration of this elementary monetary holding is $= t_R - t_A$.

Therefore, a mathematical expression of an elementary monetary holding $h(t)$ is the integral of its acquisition transaction at time t_A, and its relinquishing transaction at time t_R, provided that the acquisition transaction is accounted as positive, and the relinquishing transaction as negative.

$$h(t) = \int_{-\infty}^{t} \left(x_A(\tau) - x_R(\tau)\right) d\tau = \int_{-\infty}^{t} P(\tau)G(\tau)\left(x_A(\tau) - x_R(\tau)\right) d\tau \quad (8)$$

The lower limit of integration here is negative infinity. In reality, it should start from the instant of creation of the economy. However, this value is, in nearly every case, all but unknown. Therefore, the integration here, by convention, commences at $= -\infty$.

The inverse relation is also true: the time derivative of an elementary monetary holding, i.e. the velocity of the money exchanged by an agent, is equal to the time descriptors of the involved acquisition and relinquishing transactions:

$$\frac{\partial h(t)}{\partial t} = x_A(t) - x_R(t) = P(t)G(t)\left(\delta(t - t_A) - \delta(t - t_R)\right) \quad (9)$$

In case of elementary monetary holdings, the same amount of money is exchanged in both acquisition and relinquishing transactions. In the example from Fig. 2, the ownership of the amount of money X has been transferred from agent B to agent A at time instant t_A, and then from agent A to agent C at time instant t_R. We say that the amount of money X has circulated in the economy, whilst during the duration of the holding $h_A(t)$, i.e. within time interval $[t_A, t_R]$, agent A has withheld that amount of money from circulation.

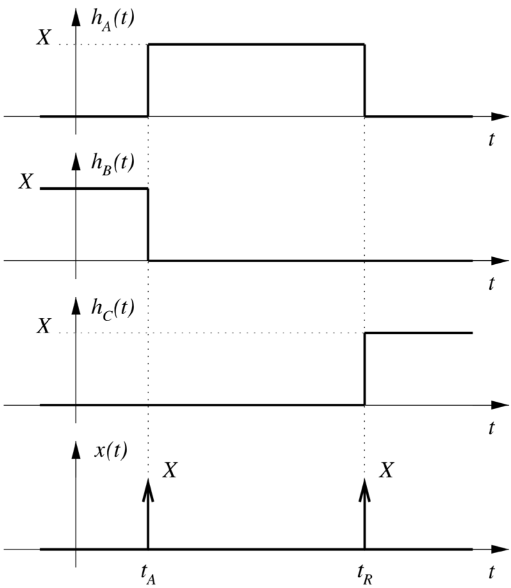

Figure 2. Elementary monetary holding and its transactions

This example shows that the role of money in an economy is that of memory. The economy as a system has remembered that agent A has provided value X to the economy, which means that the economy is obliged to provide the same value X back to agent A by some other agent at some future time.

In general, a monetary holding of an economy's agent may have a very complex form. Nevertheless, as it is clear from the example presented above, a monetary holding of an agent is necessarily a sum of the elementary monetary holdings pertaining to that agent. That is, any monetary holding is a superposition of the corresponding elementary monetary holdings, and is a piece-wise constant time function. Visually, it has a stair-like shape. Thus, a monetary holding of agent, $H^{(A)}(t)$, which is a sum of all the elementary holdings by that agent, may be represented analytically as:

$$\begin{aligned} H^{(A)}(t) &= \sum_i h_i^{(A)}(t) \\ &= \int_{-\infty}^{t} \sum_i P_i^{(A)}(\tau) G_i^{(A)}(\tau) \left(\delta\left(\tau - t_{A,i}\right) - \delta\left(\tau - t_{R,i}\right) \right) d\tau \end{aligned} \tag{10}$$

where $i \in \{$ all elementary holdings of agent A $\}$.

Dimensionally, the monetary holding $H(t)$ and the elementary monetary holding $h(t)$ both are naturally quantified in monetary units, e.g. in dollars [$]. Having shown that the time functions of the transactions forming an elementary monetary holding are produced by the time derivative of that elementary holding function, it becomes clear that the dimension of transactions $x_A(t)$ and $x_R(t)$ is money per time, e.g. dollars per second [$\$s^{-1}$].

4. Quantity of Money in an Economy

As follows from the previous section, the total quantity of money in an economy, i.e. the money supply is perceived as the total memorising capacity for values provided to the economy by the agents. This memorising capacity effectively limits the maximal level of economic activity, i.e. the number of transactions in the economy, their timing, and their total values.

An exact way to compute the quantity of money in an economy $M(t)$ at time instant t would be to sum all the monetary holdings held at that time instant by all the agents in the economy. In other words, the quantity of money at time t is the sum of all the transaction values since the inception of that economic system, and involving all the monetary holdings ever held by all the economy's agents. By using this method, the correct result is computed, but, in practice, this computation is nearly impossible to perform.

$$M(t) = \sum_j H^{(j)}(t) = \sum_j \sum_i h_i^{(j)}(t) \tag{11}$$

where $j \in$ { all agents }, and $i \in$ { all elementary holdings of agent j }

Another way to compute the quantity of money is by integrating all the transactions ever performed in the economy, provided that all the transactions that are sales are accounted as positive, whilst all the transactions that are acquisitions are accounted as negative. This is just as hopeless as the previous attempt at computing the quantity of money in an economy. Nnevertheless, it offers an opportunity to use transaction values instead of monetary holdings.

$$\begin{aligned} M(t) &= \int_{-\infty}^{t} \frac{\partial M(t)}{\partial t} d\tau \qquad (12) \\ &= \int_{-\infty}^{t} \sum_j \frac{\partial H^{(j)}(t)}{\partial t} d\tau = \int_{-\infty}^{t} \sum_j \sum_i \frac{\partial h_i^{(j)}(t)}{\partial t} d\tau \\ &= \int_{-\infty}^{t} \sum_j \sum_i P_i^{(j)}(\tau) G_i^{(j)}(\tau) \left(\delta(\tau - t_{A,i}) - \delta(\tau - t_{R,i}) \right) d\tau \end{aligned}$$

where $j \in$ { all agents }, and $i \in$ { all elementary holdings of agent j }.

Even though the exact value of the quantity of money in an economy cannot be computed directly, it may be possible to estimate this quantity within a limited interval of time $[T_1, T_2]$ by summing the values of all the transactions within this time interval.

$$M(T_1, T_2) \approx \frac{1}{2} \int_{T_1}^{T_2} \sum_j \sum_i \left| \frac{\partial h_i^{(j)}(t)}{\partial t} \right| d\tau \tag{13a}$$

$$\approx \frac{1}{2} \int_{T_1}^{T_2} \sum_j \sum_i P_i^{(j)}(\tau) G_i^{(j)}(\tau) \left| \delta(\tau - t_{A,i}) - \delta(\tau - t_{R,i}) \right| d\tau \tag{13b}$$

$$\approx \frac{1}{2} \int_{T_1}^{T_2} \sum_j \sum_i P_i^{(j)}(\tau) G_i^{(j)}(\tau) \left(\delta(\tau - t_{A,i}) + \delta(\tau - t_{R,i}) \right) d\tau$$

where $j \in \{$ all agents $\}$, and $i \in \{$ all elementary holdings of agent j $\}$.

Here, attention has to be paid to the fact that a transaction is accounted as positive for the seller, whose monetary holding it increases, whilst it is accounted as negative for the buyer, whose monetary holding that transaction decreases. This way of accounting transactions would bring to a mutual cancellation of a sell and a corresponding buy in the final sum. For this reason, the signs of the relinquishing transactions of a monetary holding are changed to positive. However, this would double the value of the estimate. Therefore, a halving of the result of the integral must take place.

Assuming that the economy is a stationary system within time interval $[T_1, T_2]$, then the result of the integration will depend only on the length of this interval. The length of this interval may be reasonably taken to be equal to the expected withholding time of money from circulation, i.e. the expected duration of elementary monetary holdings computed across all the agents. This would give an accurate estimate of $M(t)$ in this interval.

$$\begin{aligned} M &= M(T_W) = M(0, T_2 - T_1) \qquad (14) \\ &= \frac{1}{2}\int_0^{T_2-T_1} \sum_j \sum_i P_i^{(j)}(\tau) G_i^{(j)}(\tau) \left(\delta\left(\tau - t_{A,i}\right) + \delta\left(\tau - t_{R,i}\right)\right) d\tau \\ &= \frac{1}{2}\int_0^{T_2-T_1} \sum_j \sum_i P_i^{(j)}(\tau) G_i^{(j)}(\tau) \left(\delta\left(\tau - t_{A,i}\right) + \delta\left(\tau - t_{R,i}\right)\right) d\tau \end{aligned}$$

where $j \in \{$ all agents $\}$, and $i \in \{$ all elementary holdings of agent j $\}$.

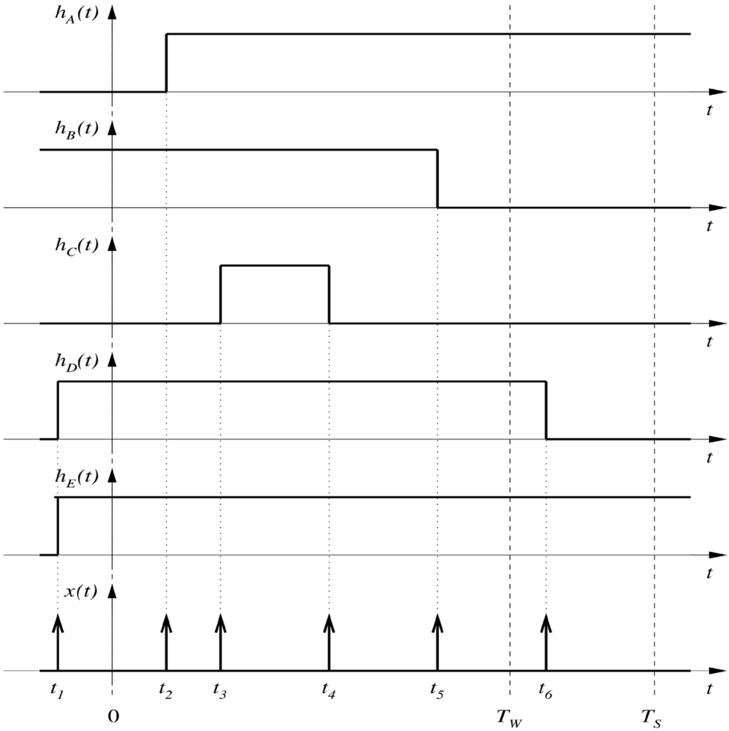

Figure 3. Types of elementary monetary holdings with respect to the integrating interval

Attention should be paid here to the possible distribution of withholding times. In fact, there are four distinct classes of monetary holdings with respect to the expected withholding time T_W. The first class of elementary monetary holdings are such holdings as $h_A(t)$ and $h_B(t)$ (see Fig. 3). The elementary monetary holdings in this class have either their acquisition transaction or their relinquishing transaction, but not both, within the integration interval $[0, T_W]$, e.g.

the transactions shown in Fig. 3 at times t_2 and t_5. Their contribution to the estimated quantity of money is correctly taken only once. The second class of elementary monetary holdings, such as $h_C(t)$, are the holdings that have both the acquisition and the relinquishing transactions within the interval of integration, e.g. the transactions at times t_3 and t_4. Only one of these transactions should be accounted for the estimation of quantity of money in the economy, but, in fact, both are. Therefore, summing the values of the transactions related to this class of elementary monetary holdings causes overestimation of the quantity of money. However, there are also elementary monetary holdings of the third class. The third class of elementary monetary holdings, such as $h_D(t)$, are the holdings that stride the interval of integration overall. The time of the acquisition transaction of such monetary holdings is before the commencement of the integration interval, i.e. $t_1 < 0$, whereas the relinquishing transaction happens at time after T_W , that is, after the end of the integration interval, but still before another future time instant T_S , i.e. $T_W < t_6 < T_S$. These transactions will not contribute to the final sum at all, and will produce a shortage in the estimate of M.

The two delimiting times, T_W and T_S, should be chosen so that the excess part and the shortage part of the estimation cancel out each other. These times depend on the concrete form of the distribution of the durations of elementary monetary holdings, so their determination has to take into account the actual behaviour of the agents in an economic system. A discussion on these matters falls beyond the scope of this work.

Finally, there are elementary monetary holdings whose duration is longer even than T_S, e.g. holding $h_E(t)$. Money of the monetary holdings of this class is deemed to be completely withheld from circulation. This money will be called the savings, and its quantity denoted by. On the other hand, money in the monetary holdings of the first three classes is deemed to be circulating in the economy. The amount of money in circulation is denoted by M.

$$M = C + S \tag{15a}$$

$$= \frac{1}{2}\int_0^{T_W} \sum_j \left|\frac{\partial H^{(j)}(t)}{\partial t}\right| dt + S = \frac{1}{2}\int_0^{T_W} \sum_j \sum_i \left|\frac{\partial h_i^{(j)}(t)}{\partial t}\right| dt + S \quad (15b)$$

where $j \in \{$ all agents $\}$, and $i \in \{$ all elementary holdings of agent j $\}$.

In a sufficiently dynamic economy, the transactions happen at ample frequency, so that the monetary holding functions, normally piecewise constant functions, approximate sufficiently well functions that have continuous time derivatives. Thus we may write:

$$\frac{\partial H(t)}{\partial t} = \sum_i \frac{\partial h_i(t)}{\partial t} = \sum_i P(t)G(t)\delta\big(t - t_{A,i}\big) - \delta\big(t - t_{R,i}\big) \quad (16)$$
$$\approx \sum_i \frac{\partial\big(P(t)G(t)\big)}{\partial t}$$

where $i \in \{$ all elementary holdings by all the agents $\}$.

For a moment, the attention is turned to the circulation part of the quantity of money. Instead of summing over all monetary holdings by all the agents, we could obtain the same result by summing over all the transactions involving all kinds of goods or services.

$$C = \frac{1}{2}\int_0^{T_W} \sum_k \left|\frac{\partial\big(P_k(t)G_k(t)\big)}{\partial t}\right| dt = \frac{1}{2}\int_0^{T_W} \sum_k P_k(t) \left|\frac{\partial G_k(t)}{\partial t}\right| dt \quad (17)$$
$$= \frac{1}{2}\int_0^{T_W} \sum_k P_k(t)V_k(t)\, dt$$

where $k \in \{$ all kinds of goods or services $\}$.

The time derivative $\frac{\partial G_k(t)}{\partial t} = V_k(t)$ is the velocity of trade of goods of kind, i.e. the quantity of goods of kind k traded per unit of time. It has been assumed that the variation of prices is much slower than the velocity of trade. Therefore the time derivative of the function of prices of goods of kind k has been neglected, $\frac{\partial P_k(t)}{\partial t} \approx 0$.

If the aggregate velocity of trade of goods and services is taken, $V(t)$, together with the price average pondered by the traded quantities, $P(t)$, and then assuming that the price levels and the traded quantities in an economy remain uniform throughout the period of estimation, then the equation of the estimated quantity of money in an economy simplifies to:

$$M = C + S = \frac{1}{2}\int_0^{T_W} P(t)V(t)dt + S = \frac{1}{2}PVT_W + S \qquad (18)$$

Finally, by using symbolic substitution $= \frac{T_W}{2}$, we obtain Irving Fisher's equation in a revised form :

$$M = C + S = PVT + S \qquad (19)$$

where M is the estimated quantity of money in an economy, having the dimension of money, e.g. dollars [\$], C is the money in circulation, with the same dimension, e.g. of dollars, [\$], S is the money in savings, also having the dimension of money, e.g. dollars, [\$], V is the aggregate quantity of traded goods or services per unit of time, i.e. the aggregate velocity of trade, also called the aggregate economic activity, having dimension of unit of goods or services per unit of time, e.g. kilograms per second $[kgs^{-1}]$, P is the price level averaged by the traded quantities of goods or services, having dimension of unit of money per unit of goods or services, e.g. dollars per kilogram $[\$kg^{-1}]$, and where T is half of the expected duration of an elementary monetary holding, i.e. half of the expected period of withholding money from circulation, with the dimension of time, e.g. seconds, $[s]$.

Essentially, this equation is the same as the original Fisher equation, however, the variables involved in this form may have a clearer interpretation, which may enable a wider and a more accurate application this extremely useful mathematical object.

A final comment on equation (19) is the relation between the level of savings, and the expected time of withholding money by the agents, expressed in the equation by T. These two quantities are not mutually independent. Naturally, an increase in the level of savings will be automatically reflected in a longer expected duration time of money withholding. Attention will be paid to this fact in the analysis performed in the next section.

5. Analysis of Monetary Inflation

In this section, the monetary or price inflation in an economy will be briefly commented. For this, the money exchange equation in its modified form (19) will be used. In this analysis, the monetary inflation is taken to mean a process of price increasing across the whole market. This process may be a consequence of the growth of the monetary aggregate, but, as it will be shown, not necessarily so.

For example, if the growth of the quantity of money were absorbed entirely by the savings (deposits), and if the quantity of money in circulation remained constant, there is no reason for the prices to change. Moreover, even if the growth of the monetary aggregate is directed entirely to the circulation part, the consequence need not to be reflected in an increase of prices, because it may be compensated by an increase in the velocity of trade, that is to say, by an augmented economic activity.

On the other hand, inflation can be caused even without an increase in the quantity of money available in the economy. Prices could grow to compensate a decrease in the velocity of trade. For example, a shortage of tradable goods may result in their higher prices. Also, a decrease in the expected time of withholding of the money, and, related to it, a decrease in the level of savings, may also cause an increase in prices without any increase in the monetary aggregate.

All the mentioned causes of inflation indeed may occur concurrently, and their effects may be combined in various ways. Thus, uncovering the causes of price inflation in each particular case is a complex task. However, using the equation of the quantity of money in its alternative form (19) may help elucidating the causes and the consequences of inflation and other monetary phenomena in an economy.

In order to facilitate the analysis of the phenomenon of inflation using the equation (19), the following Δ "delta" notation will be used. In this notation, ΔX in an equation will mean a small positive change of quantity. This small quantity is unspecified, but is of such value that the nature of the equation is preserved. If only one quantity in an equation would be modified, the equality of the sides of that equation would be necessarily invalidated. Thus, there always must be at least two quantities with modified values. Then, an additional condition is that the values of the delta changes are such that the equation sides' equality is maintained.

The working of delta notation is briefly explained with some simple examples. If the original equation is $X + Y = Z$, modifying just one quantity cancels the equality of its left and right hand sides $(X + \Delta X) + Y \neq Z$. In order to preserve the equality, two quantities has to be modified at the same time. We may have the following case $(X + \Delta X) + (Y - \Delta Y) = Z$. In this case, it is clear that the relation between the small changes of quantities X and Y must be such that $X = \Delta Y$. Another illustrative case is equation of form $Y = Z$. Letting both of the quantities on the left hand side vary, we have a modified equation $(X + \Delta X)(Y - \Delta Y) = Z$. In this case, in order to keep the validity of the equation, the relation between ΔX and ΔY must be $\Delta Y = Y\Delta X$, where, as quantities ΔX and ΔY are small, their product may be neglected, $\Delta X\Delta Y \approx 0$.

Now the phenomenon of monetary inflation will be analyzed using equation (19) with the help of the introduced Δ symbolics.

First, there is a conventional inflation: the inflation caused by the increased supply of money to the economy. This may be represented by equation (19) with corresponding delta modified values. This is how a monetary inflation is usually interpreted.

$$M + \Delta M = (P + \Delta P)VT + S \tag{20}$$

Next, a monetary inflation may happen even when there is no increase of the amount of money available in the economy. One possible cause of this kind of inflation may be a reduced velocity of transactions. This may happen when the amount of goods on offer in the economy is reduced, that is, when there is a crisis of production. Even though the monetary aggregate remains constant, the prices increase over the board.

$$M = (P + \Delta P)(V - \Delta V)T + S \tag{21}$$

An increased spending of the savings may cause another cause of the overall increase of prices. As it has been explained above, this also implies a shortening the withholding time of the money in circulation. In simple words, people spend more, perhaps because there is an expectation of a crisis, with anticipated shortages of goods.

$$M = (P + \Delta P)V(T - \Delta T) + (S - \Delta S) \tag{22}$$

Now, a possible case of a false equation is also mentioned. The monetary inflation, as explained above, is an overall increase of prices. In certain cases, an adjustment of buyers' preferences may happen, which results in increased prices of some kind of goods, but with decreased prices of other goods. In a simplified case of an economy with only two kinds of goods, this may be described by the following equation corresponding to this case. Even though price P_1 has been increased, price P_2 has been concurrently decreased, and neither of these events needs to be reflected in the level of the activity in the economy, nor needs the quantity of money in the economy to change. This is not inflation, but a simple adjustment of the preferences in an economy.

$$M = \big((P_1 + \Delta P_1)V_1 + (P_2 - \Delta P_2)V_2\big)T + S \tag{23}$$

Further, equation (19) reveals that not every increase in the supply of money need necessarily cause the inflation.

For example, there may be a case of a growing economy, where the velocity of transactions is being increased, without an increase in prices. This may happen due to an increased production combined with an increased level of absorption of goods and services by the market. In order to facilitate the increased trade, the supply of money ought to be increased, without causing inflation.

$$M + \Delta M = P(V + \Delta V)T + S \tag{24}$$

Yet another possibility, purely theoretical as it may be, is an increase in the preference for savings amongst the agents. This means that the quantity of money in savings is being increased, and, also, that the expected time of withholding money is being lengthened. In this case, if there is no augmentation of the money supply, the amount of money in circulation will be reduced, which may not be a desired solution. Therefore, the quantity of money in the economy may be increased, whilst both the prices and the velocity of trade may remain unchanged.

$$M + \Delta M = PV(T + \Delta T) + (S + \Delta S) \tag{25}$$

Of course, the reality of the monetary phenomena in an economy may be vastly more complex than the cases presented above. Changes may occur in more than just two quantities in equation (19). Indeed, all the quantities may vary simultaneously. Still, having a correct tool for interpreting such phenomena, e.g. Fisher's equation in its alternative form, may enable monetary and economic authorities to resolve possible negative outcomes by applying regulating measures.

Concluding Remarks

In this article, an alternative form of the Fisher's equation of quantity of money has been derived. This derivation has been justified by imprecision of this equation in its original form with respect to the dimensional interpretation of the quantities involved in it. In the process of its derivation, a clarification of the functioning of an economy from monetary point of view has been exposed. Then, an unexpected and new explanation of the role of money in a market economy as a memorising device has been offered. In this new form of the Fisher's equation, a distinction has been made between the money in circulation and the money in deposits i.e. the savings. Some interesting consequences for an economic system functioning have been inferred from this new form of the quantity of money equation. In particular, it has been shown that there are three possible causes of price inflation, and that not every increase in money supply need necessarily bring an increase in prices.

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ПОЈАШЊЕЊЕ ФИШЕРОВЕ ЈЕДНАЧИНЕ

Апстракт
Фишерова једначина, или једначина размене, повезује количину новца са ценама и економском и новчаном активношћу у економском систему. И поред своје неспорне тачности, ова једначина у свом уобичајеном облику исказује непрецизности у значењу димензија величина које повезује. Лакша интерпретација и примена ове једначине основни су мотиви за предузимање рада на овој теми. У овом раду, Фишерова једначина анализирана је димензионо, и показане

су тешкоће у њеној инерпретацији. У наставку, описано је функционисање економије, и анализирана динамика тока новца. Показано је да за сваког учесника у економском систему постоји временска функција расположивог новца, и да се ова функција може разложити на математички једноставније функције, које се мењају кроз трговинску размену. Један од закључака овог разматрања економије и протока новца јесте тај да је основна улога новца у економском систему памћење величине доприноса учесника. На основу урађене анализе, изведена је Фишерова једначина из основних математичких поставки, при чему се обраћа пажња на интерпретацију и димензиону анализу величина које се у овој једначини појављују. Како би се једначина учинила примењивијом, у сложеној једначини инфинитезималног рачуна извршена су оправдана упрошћавања. Резултат је Фишерова једначина у новом облику, коју је могуће применити на проблеме економије и новчане токове. Употреба ове једначине у њеном новом облику показана је кратком али довољно тачном анализом узрока настанка инфлације и могућих последица повећања количине новца у оптицају..

Кључне речи: *количина новца у оптицају, Фишерова једначина, монетарна економија.*

JEL klasifikacija: B16, E31, E40